MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

PCL0016 – CALCULUS

(ALL Sections/Groups)

6 MARCH 2020
9:00 A.M - 11:00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages including cover page with SIX questions.
2. Attempt **ALL SIX** questions and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

Question 1 (20 marks)

(a) Evaluate the following limits. (**Do not use** L'Hôpital's Rule):

(i) $\lim_{x\to -3}\dfrac{x^2-9}{x^2+2x-3}$. [3 marks]

(ii) $\lim_{x\to 0}\dfrac{1-\sqrt{1+x^2}}{x}$. [5 marks]

(iii) $\lim_{x\to 1}\left(\dfrac{1}{x-1}+\dfrac{1}{x^2-3x+2}\right)$. [5 marks]

(b) Find the value of m so that the given function below is continuous on $(-\infty,\infty)$. State the requirement of continuity that is used to solve the value of m.

$$f(x)=\begin{cases}\dfrac{4\sin mx}{x}, & x\neq 0\\ x+m^2+4, & x=0\end{cases}$$

[7 marks]

Question 2 (15 marks)

(a) Evaluate $\dfrac{dy}{dx}$ for the following functions:

(i) $y=x^4-\dfrac{1}{x^2}+\pi e^x-4$. [2 marks]

(ii) $y=x\ln x+\dfrac{\cos x}{e^x+2}$. [3 marks]

(iii) $y=\tan^2\left(\ln\left|3x^2\right|-\sqrt{x}\right)$. [3 marks]

(b) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ of $y=\dfrac{x^3\sqrt{x^4-2}}{(x+1)^2(3x-2)^4}$. [7 marks]

Continued.....

Question 3 (15 marks)

(a) Given a function $f(x)=(x+3)^3(2x-1)^2$ on the interval $[-3,2]$. Find the critical numbers and determine the absolute maximum and minimum values. [9 marks]

(b) The motion of a feather dropped on the moon is following the equation $x(t)=4t^3+2t^2-3t+6$. The time $t \geq 0$ is measured in seconds and $x(t)$ is measured in meters.

(i) Find the velocity and acceleration of the feather after $5s$. [4 marks]

(ii) How far from the origin is the feather before it is dropped on the moon? [2 marks]

Question 4 (15 marks)

(a) Evaluate the following integrals:

(i) $\int \frac{(x+4)\left(1-x^{\frac{1}{2}}\right)}{x^{\frac{1}{2}}}dx$ [3 marks]

(ii) $\int x^6 \ln x dx$ using integration by parts. [3 marks]

(iii) $\int_4^{12} \frac{\left(\sqrt{x}+2\right)^3}{\sqrt{x}}dx$ using integration by substitution. [4 marks]

(b) Given $f(x)=\frac{x+12}{x^2+3x-18}$. Evaluate $\int f(x)dx$ using partial fraction decomposition. [5 marks]

Continued.....

Question 5 (15 marks)

(a) A shaded region is bounded by $y^2 = x+4$, $y=8$ and y-axis as shown in **Figure 1**.

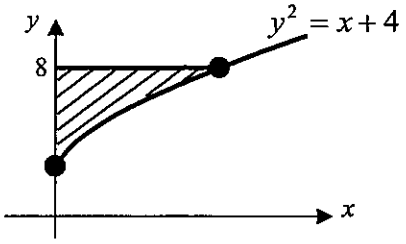

Figure 1

(i) Find the area of the shaded region. [4 marks]

(ii) Calculate the volume generated when the shaded region is revolved about the x-axis. [6 marks]

(b) Find the possible values of b such that the average value of $f(x) = 5+8x-3x^2$ on the interval $[0,b]$ is equal to 8. [5 marks]

Question 6 (20 marks)

(a) Solve the following differential equations:

(i) $\dfrac{dy}{dx} = \dfrac{x\sin x}{y}$. [5 marks]

(ii) $2y''+5y'-3y=0$, $y(0)=-3$ and $y'(0)=11$ [10 marks]

(b) Find the solution to the linear differential equation $xy'+2y = x^2 - x+1$. [5 marks]

End of Paper